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MAGNETOHYDRODYNAMIC SHOCK WAVE IN A COLLISION-FREE PLASMA

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RESEARCH REPORT 85

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ABSTRACT

In high temperature low density plasmas collisional relaxation becomes slow compared to other characteristic times (collision free plasma). It seems likely that under such circumstances more powerful dissipative mechanisms would appear and the understanding of these mechanisms is basic to the treatment of containment and flow problems. It is known that shock waves propagating perpendicular to a magnetic field can be much thinner than a mean free path, which implies that more powerful dissipative mechanisms must exist. ~~This paper is~~ An attempt is made to identify the dissipative mechanisms operative in a shock wave with randomized magnetohydrodynamic waves of large amplitude. The entropy production process is the scattering of waves on waves. The typical "wave mean free path" is comparable to an ion Larmor radius inside a shock front. The short mean free path for this scattering process implies that continuum magnetohydrodynamics can be applied in many cases even when the interparticle mean free path is quite large. Both the shock thickness and its dependence on the Alfvén Mach number obtained in this manner are in agreement with "MAST" shock tube experiments.

Introduction

The basic dissipative mechanisms which occur in a plasma are of fundamental importance in determining the behavior of the plasma. There has been considerable evidence from high temperature plasma experiments that the dissipation rates are much more rapid than would be expected on the basis of interparticle collisions. In these cases, this results in a rapid diffusion of the plasma to the walls. Similar rapid diffusion processes would be of interest in many astrophysical situations such as the penetration of gas streams from the sun into the earth's magnetic field.

In terms of the orientation of this conference towards continuum treatments of magnetohydrodynamics, the existence of rapid dissipation mechanisms implies that continuum treatments (in the aerodynamic sense) are applicable under gas conditions where, at first sight, one might not expect them to be. The fundamental basis of continuum theories is that a dissipative randomizing process exists on a scale small compared to the characteristic size of the flow field. Rapid dissipation implies a mean free path for randomization which is smaller than one would have anticipated.

In Reference 1, a suggested classification of different regions in plasmas was made on the basis of some of the characteristic lengths in

the plasmas. Regions were defined in terms of temperature and density where one might expect the plasma to have different fundamental characteristics. A map representing this classification is reproduced in Fig. 1 in this paper. We will investigate in particular the dissipation mechanisms occurring in the M region. This region is defined by three principal boundaries: (1) that the mean free path for interparticle collisions defined by the coulomb cross-section is larger than the ion Larmor radius; (2) that the electron thermal motion be non-relativistic; and (3) that the ion Larmor radius be less than the characteristic size of the flow field, which in Fig. 1 has been arbitrarily chosen as 1 cm. In order to emphasize magnetohydrodynamic phenomena, this map was drawn under the assumption that the magnetic energy density was equal to the particle pressure.

As can be seen from Fig. 1, this region encompasses many high temperature plasma experiments, and, if the appropriate change in the length scale is also made, includes much of the interplanetary and interstellar gas. Further, it has been shown that the dissipation mechanism which occurs in this region must be more rapid than that associated with ordinary interparticle collisions.[2]

Exhibition of Dissipation by Shock Waves

The problem which seems best suited for studying dissipation mechanisms is the investigation of the structure of a shock wave. In the first place, the conservation equations require the existence of a dissipation mechanism in the shock front. Secondly, a first integral of the conservation equations in the shock wave can be obtained readily. Thus,

considerable knowledge of conditions inside the shock wave is available and is independent of the dissipation mechanism.

For simplicity, we shall consider a shock wave moving perpendicular to the magnetic field lines. In order to have a well-defined gas state ahead of the shock wave, we will assume the gas temperature there to be zero. Other choices would, of course, be possible. However, this case is most easily approximated experimentally. The minimum velocity for such a shock wave is the small amplitude disturbance speed through the cold plasma, and this is the Alfvén speed. We will consider shock waves of moderate strength, i.e., gases where the ratio of the supersonic stream velocity to the Alfvén speed (the Alfvén Mach number) is between 1.5 and 3.

In many astrophysical or laboratory cases, shock waves are formed by the steepening of gradual compression fronts. As the steepness of the front increases, diffusion processes become important and at some steepness can transfer sufficient momentum and energy, and produce sufficient entropy so that a steady state shock profile is attained. Conversely, if a shock is formed from a very steep pulse and diffusion processes exist which can act over a wider range than the pulse, these processes would tend to broaden the shock wave. One would therefore expect that the diffusion process which can act at the minimum steepness, i.e., at the longest range, will be the one which controls the shock structure, provided that it can produce sufficient dissipation. Although this suggestion has by no means been proved, it is sufficiently plausible that it will be used in a qualitative sense and will be referred to as the thickest shock hypothesis.

In an un-ionized monatomic gas, the steepening process is inhibited and a steady state shock structure is obtained when the shock thickness is of the order of the mean free path for interparticle collisions. The argument in Reference 2 for rapid dissipation rates is based on the fact that for a plasma under the conditions mentioned above, the steepening of the pulse is not inhibited at this point. Physically, this can be explained in terms of the heat conduction and viscosity coefficients being appreciably decreased when the particles go through many gyro orbits between collisions. The next longest basic dimension of the plasma which could have an effect on limiting the shock thickness is the characteristic ion Larmor radius.[3]

In this paper, we will attempt to show that randomized magnetohydrodynamic waves can provide the dissipation necessary when the shock thickness is of this order of magnitude. It has been suggested by Kahn[4] and by Parker[5] that a dissipative mechanism can be found in terms of plasma oscillations which leads to a shock thickness of the order of the Debye length. Gardner, et al[6] have suggested that a permanent shock structure can be found from a series of pulses whose steepness is associated with the characteristic electron Larmor radius.[3] Both of these thicknesses are appreciably smaller than the one suggested here and one would therefore expect that these shock structures would be broadened out by the mechanisms suggested here.

Proposed Shock Model

The fundamental model for the shock wave which is proposed here is that in the shock front the thermal energy which is produced is initially

almost all invested in a random distribution of magnetohydrodynamic waves. That is to say, the dissipation in the shock wave excites only the relatively few degrees of freedom of the gas which are associated with magneto-hydrodynamic waves. The much larger number of degrees of freedom corresponding to the actual thermal velocities of particles are not appreciably excited in the shock front itself. The relaxation from wave energy to a Maxwelliam distribution where the bulk of the thermal energy is associated with individual particle motions is a slower process and takes place behind the shock front, i.e., behind the region where most of the density change occurs.

We will now view the gas from a scale that is sufficiently gross so that the random distribution of waves may be thought of as the microscopic structure of the plasma. The interactions between waves, which result from non-linear terms, will then increase the randomness in the waves and correspond to an increase in entropy.[7] The basic dissipative mechanism is then the scattering of waves on waves.

That this dissipation mechanism is more rapid than interparticle collisions is suggested by a comparison of the scattering lengths or mean free paths for the two cases. If the wave amplitudes are sufficient to change the gas state appreciably from its average value, the propagation velocity of a single wave will be changed appreciably by the other waves present. In going a wavelength of the disturbing field a wave of comparable wavelength will then have its phase changed appreciably. This corresponds to a scattering of the wave. In the presence of large amplitude waves, the mean free path of the waves is therefore of the order of the wavelength.

In a sufficiently high temperature plasma, we may expect the interparticle mean free path to be appreciably larger than the typical wavelengths which can arise.

An alternate description of the entropy production may be obtained if one views the waves as groups of particles which tend to move coherently. The interaction of two such groups will be stronger than the interaction of individual particles and, therefore, these groups will have a shorter mean free path for collisions with each other than the individual particles in the gas. The entropy associated with the motion of these groups will, therefore, be increased more rapidly than the entropy associated with individual particles.

In developing the transport coefficients for the plasma, the waves may be considered analogous to the fundamental particles of kinetic theory of gases. A "kinetic theory" of the plasma may be developed in terms of the wave motions and their collisions with each other.

In order to justify this model, and to estimate the shock thickness, we will first demonstrate that a mechanism exists for building up these wave amplitudes; secondly, describe the particular waves which are selected as being most important; and thirdly, estimate the interactions between these waves that determine the mean free path for the waves. The mean free path will then be used to estimate the shock thickness.

Wave Growth

If a wave packet is superposed on a compression front, its energy will increase with time. This can be seen in the following way. A wave packet will exert a pressure of the order of magnitude of the wave energy

per unit volume on the surrounding medium. If the surrounding medium is undergoing a compression, the work done against the pressure exerted by the wave packet will appear as increased energy of the wave packet. In the case of a one-dimensional compression front propagating in the x direction, this work is given by $p_{xx} \frac{du}{dx}$ per unit time where p_{xx} is the stress exerted by the wave packet in the x direction across a plane perpendicular to x and u is the flow velocity. It can be seen that if a wave packet spends a time in the shock front equal to the particle time in the shock front, then the ratio of its energy across the shock is similar to that experienced by a gas undergoing a similar isentropic compression. If, however, a wave packet spends a long time in the shock front, its energy can grow indefinitely.

In order to achieve a wave growth which is larger than the wave growth corresponding to an isentropic compression, the waves must move such that they spend more time in the shock front than the gas particles, i.e., they must have a drift velocity relative to the gas towards the upstream side of the shock wave. Since in a shock wave of moderate strength, the change in wave energy that is required is much more than the isentropic change, the velocities of the waves relative to the fluid must be of the order of magnitude of the fluid velocity itself. Therefore, if waves exist which can move at such a velocity, they will be amplified by a sufficient amount.

It is important to note that since a wave proceeding in an arbitrary direction will exert a pressure in the x direction, this amplification process can operate for waves of arbitrary orientation provided that there is sufficient velocity in the x direction.

Selection of Important Waves

The characteristics of the linear waves which can propagate in a plasma with zero particle temperature can be obtained by the usual small amplitude perturbation procedures. For our case, we can simplify this to some extent on the basis of the thickest shock hypothesis. This hypothesis implies that we are interested in the longest wavelength waves which can produce a steady state shock structure. Consequently, we may neglect terms associated with charge separation, since these have a characteristic length of the order of the Debye length. Further, we may consider the electron Larmor radius as being small compared to the wavelength. The above two assumptions correspond to assuming charge neutrality and taking the electron mass to be zero. With these approximations, the linearized equations become

$$\underset{\sim}{E} + \frac{\underset{\sim}{u}_e \times \underset{\sim}{H}_o}{c} = 0, \qquad m_i \frac{\partial \underset{\sim}{u}_i}{\partial t} - e\left(\underset{\sim}{E} + \frac{\underset{\sim}{u}_i \times \underset{\sim}{H}_o}{c}\right) = 0,$$

$$\nabla \times \underset{\sim}{H} = 4\pi \frac{N_o e}{c}\left(\underset{\sim}{u}_i - \underset{\sim}{u}_e\right), \qquad \nabla \times \underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{H}}{\partial t}, \tag{1}$$

where $\underset{\sim}{E}$ is the electric field, $\underset{\sim}{H}$ is the magnetic field, $\underset{\sim}{u}_e$ and $\underset{\sim}{u}_i$ are the electron and ion velocities, e and m_i are the ion charge and mass, c is the velocity of light and the subscript o refers to the unperturbed plasma conditions. The resulting dispersion relation is

$$\left(\frac{\omega}{\omega_i}\right)^4 - \left(\frac{\omega}{\omega_i}\right)^2 (r_i k)^2 \left[1 + \cos^2\theta + (kr_i)^2 \cos^2\theta\right] + (r_i k)^4 \cos^2\theta = 0, \tag{2}$$

where ω is the frequency, ω_i is to the ion cyclotron frequency, k is the wave number, θ is the angle between the wave normal and the unperturbed magnetic field, and r_i is defined in footnote 3. Since this dispersion relation is quartic in ω, there are two wave modes possible. For small values of $r_i k \cos\theta$, these reduce to one wave moving at the Alfvén speed in all directions and a second slower wave which moves at the Alfvén speed based on the magnetic field component parallel to the wave vector. As the wave number increases the phase velocity of the faster of these waves increases, while that of the slower wave decreases.

In order to satisfy the criterion derived in the last section, we must find waves whose group velocity is comparable to the flow velocity in the shock. Since the shock wave must move faster than the Alfvén speed ahead, we must look for waves whose group velocity, perpendicular to the magnetic field, is greater than the Alfvén speed. In Fig. 2, this component of the group velocity is plotted for the fast mode as a function of the absolute value of the wave number, with the direction of the wave vector chosen to maximize this component of the group velocity. As can be seen, the fast wave can satisfy the criterion of moving faster than the Alfvén speed. The slow wave never has a group velocity perpendicular to the field of more than half the Alfvén speed.

The dominant waves which will be present in the shock wave will therefore be the fast waves described above which have a group velocity perpendicular to the magnetic field comparable to the flow velocity ahead of the shock wave. (The ratios of group velocity to Alfvén speed must be comparable with the Alfvén Mach number of the shock). The waves which exist should be thought of as a distribution of waves in wave number space surrounding this general area.

The general characteristics of waves in this area are that the energy of the wave is mostly in magnetic energy and that the waves are essentially circularly polarized. This may be understood physically in the following way: when the frequency becomes large compared to the ion cyclotron frequency, the ions cannot follow the wave motion and therefore the kinetic energy of the ions will be small. Since magnetic energy is not interchanged with kinetic energy, the magnetic energy must remain constant through the wave, and this can be accomplished only by a circularly polarized magnetic vector. The direction of polarization is opposite to the rotation of the ions about the zero order magnetic field. For the slow wave, the polarization was predominantly in the opposite direction, and hence, these waves in the limit of large wave numbers correspond to large kinetic energies. If a small but finite particle temperature had been considered, the slow wave would be strongly damped for large values of $r_i k$.

Wave Mean Free Path

The interactions between waves may be estimated by considering the motion of one wave through the disturbed field produced by the other waves that are present. A wave will be appreciably altered in amplitude and direction when its phase has been changed by unity (or its "optical path" changed by $1/k$) due to the disturbing waves. If we approximate Eq. (2) by neglecting the angular dependence of the dispersion relation and assuming $r_i k \cos \theta \gg 1$, it may be written as

$$\omega = \frac{eH_o}{mc} \quad r_i^2 k^2 \ . \tag{3}$$

Since, as pointed out previously, the particle velocities associated with the wave are small when $r_i k \cos\theta \gg 1$, the density fluctuations may also be neglected. Hence, r_i in the above equation may be considered as a constant, i.e., it is unaffected by the perturbing wave field. At constant frequency the change in wave number is then related to the perturbing magnetic field ΔH_o by

$$\frac{\Delta H_o}{H_o} = - \frac{2\Delta k}{k} . \qquad (4)$$

The perturbing field of the other waves will be coherent over a distance of the order of the reciprocal of the mean wave number, i.e., $\frac{1}{k_m}$. In going this distance, the phase of the wave therefore changes by an amount $\frac{\Delta H_o}{2H_o} \frac{k}{k_m}$. The phase changes by a random walk process with steps of this amount each time the wave goes a distance $\frac{1}{k_m}$. The length λ required to obtain an r. m. s. phase change of unity (the mean free path) is therefore given by setting the product of the phase change per step and the square root of the number of steps λk_m equal to unity:

$$\lambda = \frac{4}{\beta k} \frac{k}{k_m} \approx \frac{4}{\beta k} . \qquad (5)$$

Here $\beta = \left(\frac{\Delta H_o}{H_o}\right)^2$ is the ratio of the average wave energy to the unperturbed field energy. As indicated, we will set $k_m = k$ thus assuming that only waves grouped about a single value of k have appreciable energy.

The assumption which was made above that the disturbing field is changed in a random fashion in a distance $\frac{1}{k}$ assumes that there are

no correlations between the waves. This may be justified by an argument analogous to the one used in kinetic theory for neglecting two particle correlations. The wave field may be divided up into wave packets whose dimensions are of the order of $\frac{1}{k}$. If the mean free path for the waves is appreciably larger than this wave packet size, then on consecutive collisions a wave packet will interact with a completely different set of wave packets. Therefore, on any collision, a wave packet will interact with wave packets with which it has not interacted before, and it is reasonable to assume that no correlation exists. If, on the other hand, the mean free path were less than the wave packet size, a wave would continue to interact with the same neighbors, and therefore, strong correlations would be expected. This argument is completely analogous to the kinetic theory argument, that when the mean free path becomes of the order of magnitude particle size, i.e., when one goes from the gas to the liquid phase, one expects correlations to appear. In our case, this criterion is that $\lambda k = \frac{4}{\beta} \gg 1$.

This requires that β be relatively small. One would therefore not expect the theory developed here to apply directly to shock waves with large Alfvén Mach numbers where β becomes large.[8]

The above estimate of the magnitude of wave interactions is, at best, a rough description of the interactions. To be more precise, one should distinguish between at least three types of interactions. These may be described (1) as collisions which appreciably change the magnitude of the wave vector (2) as collisions which change the direction of the wave vector and (3) as collisions which produce a net transfer of momentum from momentum of the wave field to the fluid itself. For the purposes of

the present treatment, we will not make any distinction between these three processes, although for any more refined treatment, this would be necessary.[9]

Estimate of Shock Thickness

The conservation equations throughout the shock wave may be written as

$$
\begin{aligned}
&E_y = E_{y_1} \\
&uH_z = u_1 H_{z_1} \\
&\rho u = \rho_1 u_1 \\
&\rho u^2 + \frac{H_z^2}{8\pi} + p + \tau = \rho_1 u_1^2 + \frac{H_{z_1}^{\;2}}{8\pi} \\
&\frac{1}{2}\rho u^3 + \frac{uH_z^{\;2}}{4\pi} + 4\,pu + \tau u + q = \frac{1}{2}\rho_1 u_1^{\;3} + \frac{u_1 H_{z_1}^{\;2}}{4\pi} \, ,
\end{aligned}
\tag{6}
$$

where the gas velocity has been assumed in the x direction and the magnetic field in the z direction. The gas density and velocity are denoted by ρ and u respectively, while H_z is the average local magnetic field, E_y is the average local y component of electric field and the subscript 1 refers to conditions ahead of the shock. The pressure, viscous stress and heat flux associated with the waves are denoted by p, τ and q. The second equation follows from the fact that, since the wavelengths present are much greater than the electron larmor radius, the electrons move adiabatically (corresponding to infinite conductivity) through the shock wave and therefore have a drift velocity $\dfrac{E_y}{H_z}$.

Since there can be no net current in the flow direction, the mean velocity of the ions must be the same. The energy equation has been written in terms of a ratio of specific heats of 4/3. This is the appropriate ratio of specific heats for thermal energy invested in waves which can propagate in any of three directions.

If a relation is given between the heat flux and the viscous stress, the above equations may be solved to give all of the properties inside the shock front in terms of one parameter, for example, the ratio of the velocity to the supersonic stream velocity. This relation, which is the Prandtl number in ordinary aerodynamics, is determined by the ratio of the mean free paths for momentum transfer to the fluid and for a change in the angle of the wave vector. If these are of the same order of magnitude, the two transport quantities will be roughly equal or

$$q \approx -u\tau \quad (7)$$

Using this assumption, several quantities through the shock wave have been calculated, and are shown in Figure 3. It is interesting to note that the wave pressure is appreciably higher in the center of the shock wave than it is on the subsonic side. The total pressure, i.e., the sum of the magnetic pressure and the wave pressure, rises monotonically to its downstream value. The existence of a maximum in the wave pressure is, of course, not a result of the wave picture, but the general case for a magnetohydrodynamic shock wave.

In principle, it should be possible to develop the kinetic theory picture of these waves further and estimate heat conduction and viscosity coefficients in terms of the mean free path described earlier. However,

the accuracy with which the mean free path was calculated does not seem to warrant a precise description of the kinetic theory. In parallel with the accuracy given there, it seems sufficient to assume that the shock thickness is about twice the mean free path, as it is for a strong shock wave in ordinary aerodynamics. The comparison with a strong shock wave is more appropriate than comparison with the same Mach number shock wave since the wave pressure ratio in our case is infinite, as it is for a very strong aerodynamic shock. This means that, in both cases, the entropy production in the shock must be equal to the entropy which is convected out of the shock region in the downstream gas. Since the assymmetry in the distribution function depends on the gradient per mean free path of the flow quantities, and since the rate of dissipation depends on the assymmetry squared, the total dissipation in the shock depends on the ratio of shock thickness to mean free path. Since, in both cases, comparable amounts of entropy must be produced, we would expect the ratio of shock thickness to mean free path to be roughly the same.

In terms of these assumptions and the mean free path given in Eq. (5), we may write the shock thickness as

$$L = \frac{8}{\beta_{max} k} , \tag{8}$$

where β_{max} is chosen as the maximum value of β in the shock and k is chosen to give a group velocity in the x direction equal to the supersonic flow velocity. From Fig. 2, this may be approximated by

$$kr_i = 2 \frac{u_1}{V_A} = 2 \ M_A , \tag{9}$$

where V_A is the Alfvén speed and M_A the Alfvén Mach number. The resulting shock thickness is then

$$L = \frac{4\, r_i}{\beta_{max}\, M_A} \quad . \tag{10}$$

Comparison with Experiment

Patrick[10, 11] has used a magnetic angular shock tube (MAST) to study the thickness of shock waves in this range. The shock thickness was determined from the time history of the bremsstrahlung emitted by the plasma. His results have been re-plotted in Figs. 4 and 5. By comparing the experimental shock thickness at low densities in Fig. 4 with the interparticle collision free path and with the collision shock thickness (estimated for a strong shock in the absence of a magnetic field), it may be seen that the data clearly indicate a shock thickness much smaller than could be accounted for by interparticle collisions.

The reliability of this data in predicting the actual shock thickness is open to some question. In Fig. 4, all of the measured thicknesses lie quite close to the size of the annulus which contained the gas, suggesting that geometrical effects related to the shock tube size might influence the measurement. However, attempts to see if the apparent shock thickness was due to the shock wave being tilted seemed to indicate that this was not the case. Also, some of the high Alfvén Mach number data in Fig. 5 gave thicknesses considerably less than the annulus spacing, indicating that the influence of the annulus size is small. Another questionable point is associated with the determination of the shock thickness from the oscillograms. Particularly at low Alfvén Mach numbers, the light intensity

through the shock exhibits a sharp change in slope at about half the final light intensity. The thicknesses shown correspond to the total rise time. If the detailed shape of the oscillograms is to be trusted, a definition based on the maximum slope would have indicated considerably less variation of the shock thickness with Alfvén Mach number. However, since the theoretical estimate is based on a gross overall thickness, it seems appropriate to compare it with the thickness based on the total rise time.

The dependence on density in Fig. 4 and on Alfvén Mach number in Fig. 5 is in fortuitously good agreement with the theoretical estimate. The absolute magnitude of the theoretical prediction is at best good to a factor of three. Equally valid arguments can be presented which vary the result by this amount. In particular, the apparent improvement between this prediction and the one presented previously[12] is not to be taken as a refinement of the theory.

If the apparent agreement is to be taken as a confirmation of the theoretical model, it is interesting to speculate that a similar type of kinetic theory based on wave interactions would be appropriate for other dissipative flows in plasmas. It is worth noting that since the waves which have been postulated will have some damping, thermalization of the particle motions themselves will probably also occur in a distance small compared to the interparticle mean free path.

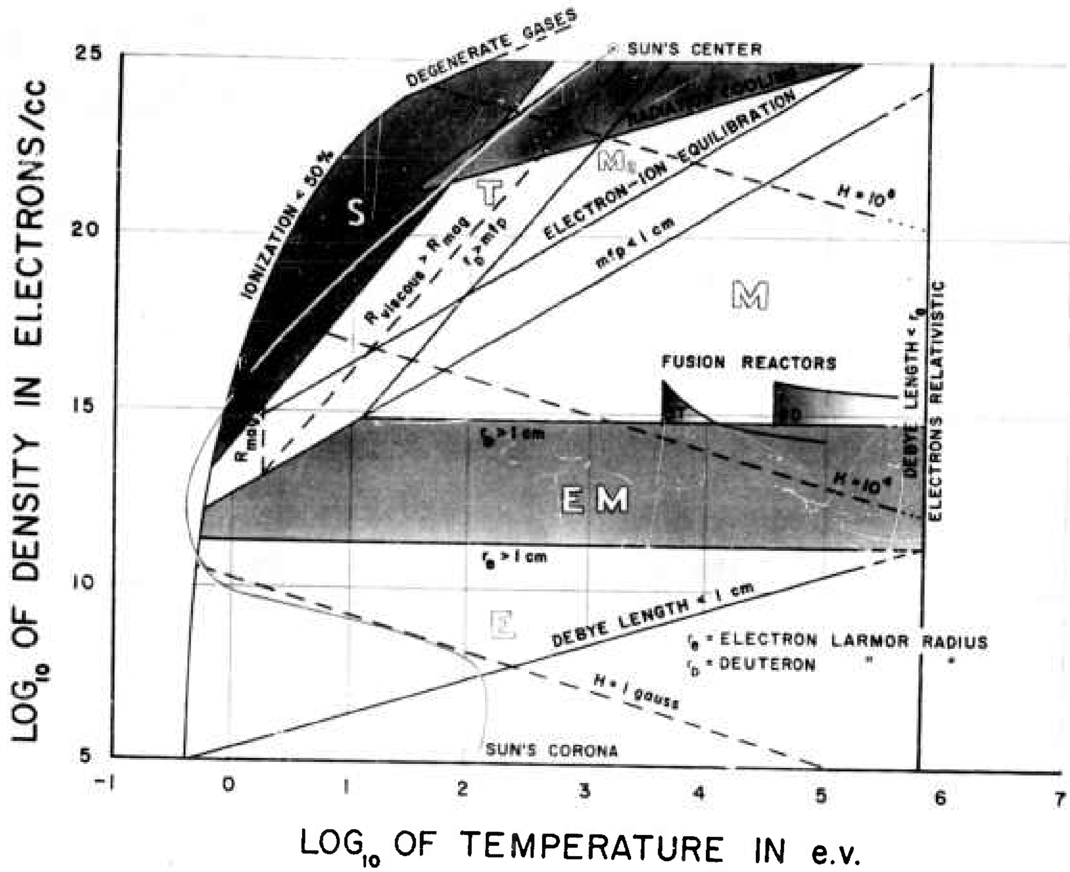

Fig. 1 Classification of different regions in plasma dynamics. In each area of different shading, one would expect appreciably different plasma characteristics. The gas pressure was assumed equal to the magnetic energy density and the ions were assumed to be deuterons.

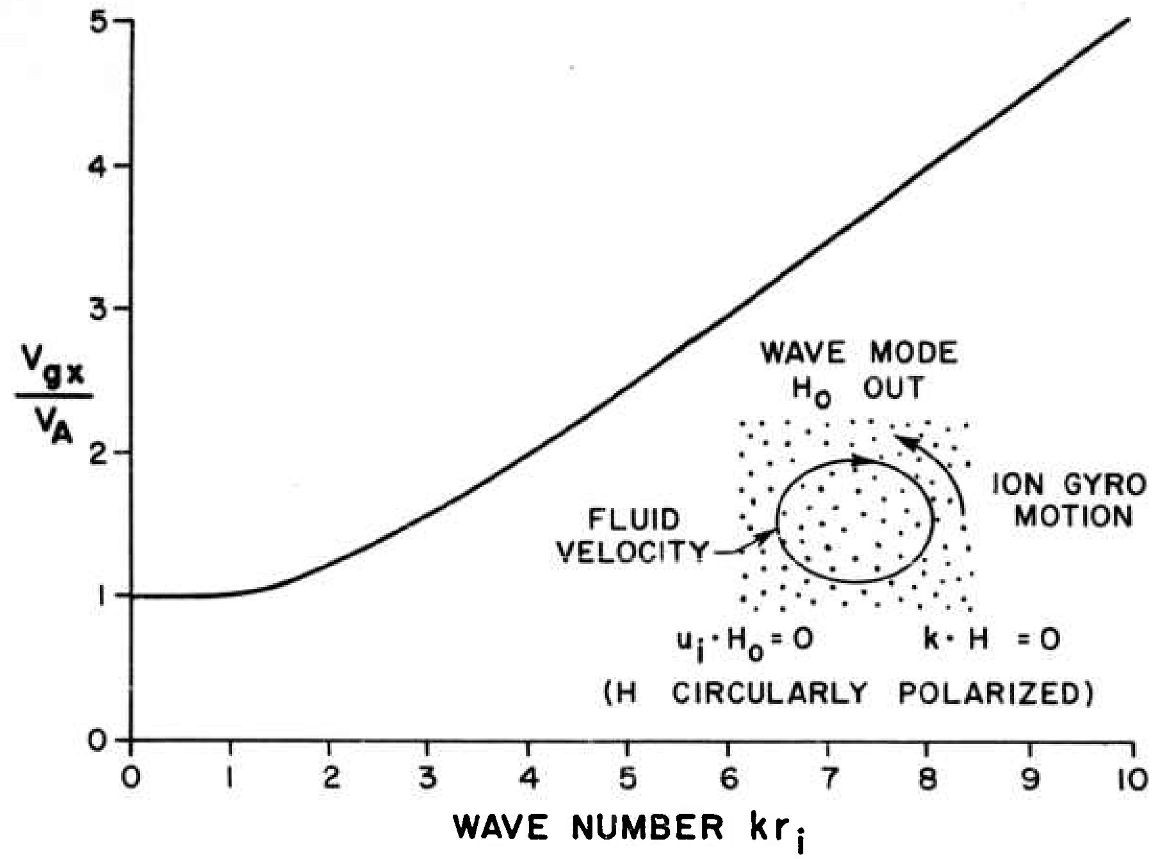

Fig. 2 The ratio of the maximum group velocity perpendicular to the magnetic field to the Alfvén speed as a function of the product of the characteristic ion Larmor radius and the magnitude of the wave number for the faster of the two magnetohydrodynamic modes in a zero temperature plasma.

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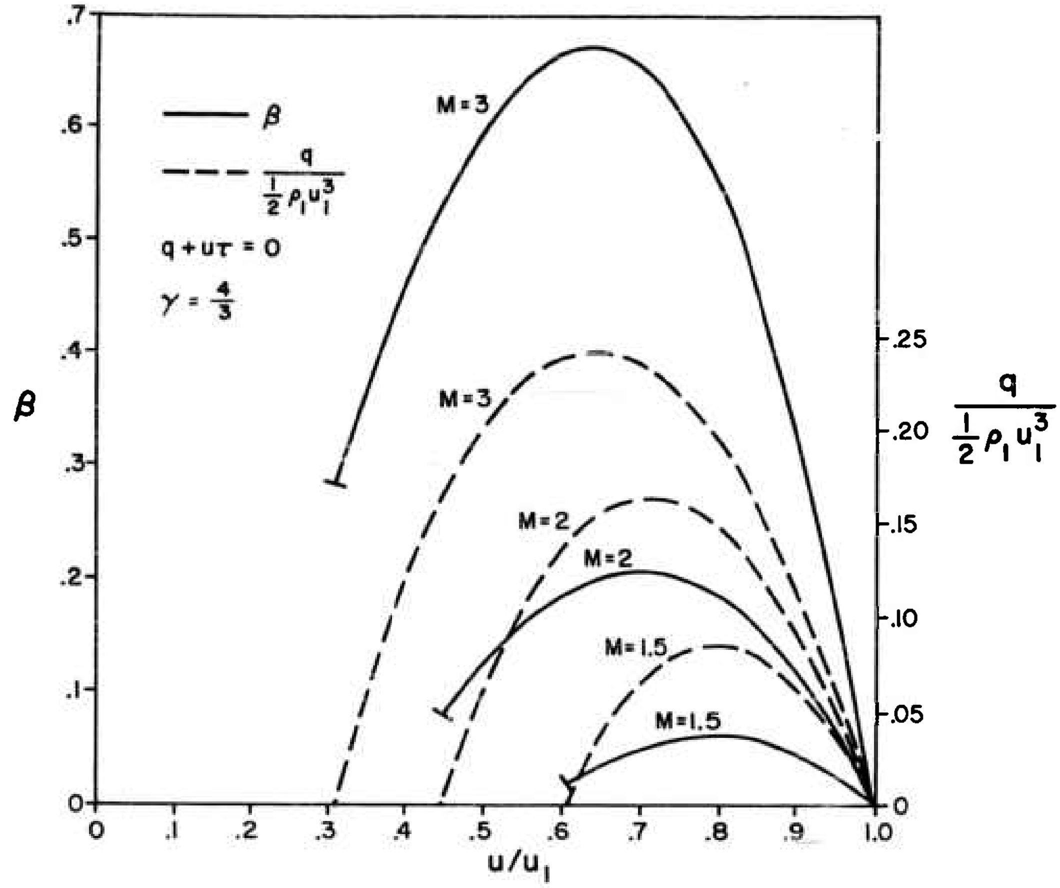

Fig. 3 The ratio of pressure to the energy density of the uniform component of the magnetic field and the ratio of heat flux to the kinetic energy flow in the supersonic stream are plotted as a function of the ratio of velocity to the supersonic stream velocity. These curves have been drawn under the assumption that the viscous and heat conduction effecis are comparable. The ratio of specific heats, denoted by γ , was taken to be 4/3. The curves of the non-dimensional pressure have been terminated at the final value corresponding to the downstream flow condition.

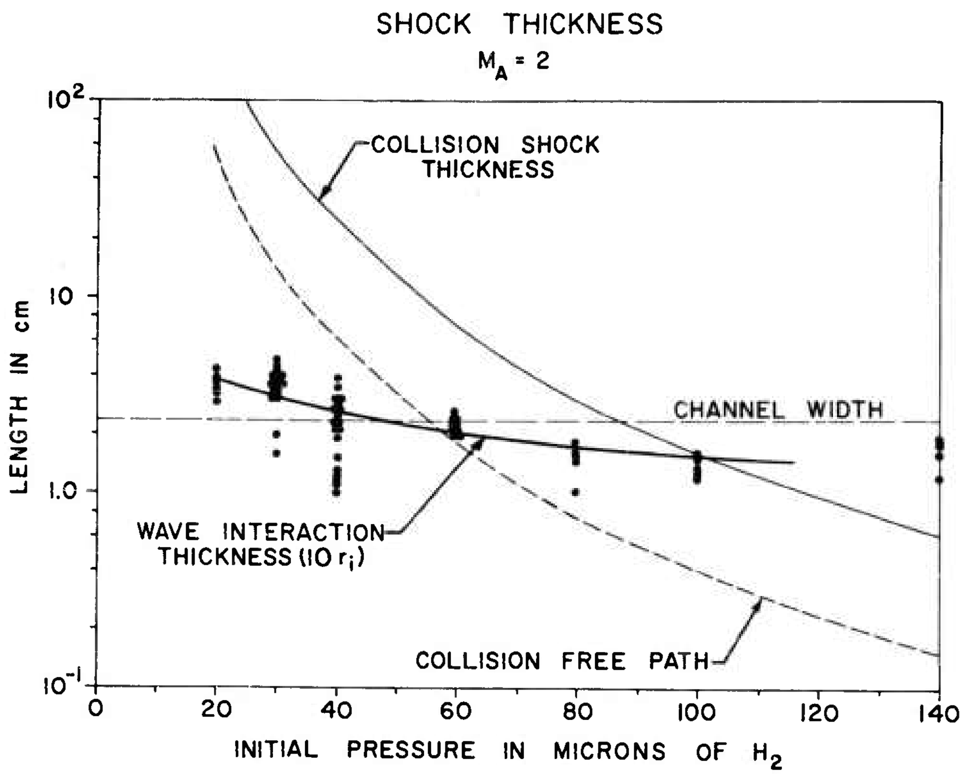

Fig. 4 Comparison of the experimental density dependence of the shock thickness with the theoretical estimate based on wave interactions. The agreement is much better than the uncertainty in the theoretical prediction, which is at least a factor of three.

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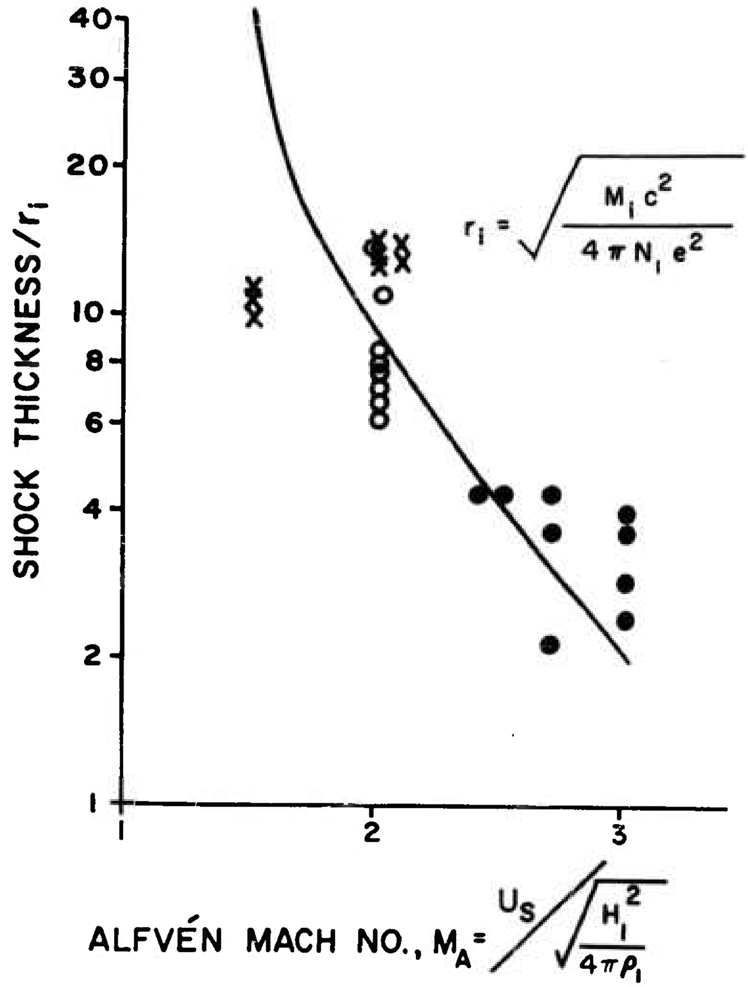

Fig. 5 Comparison of the experimental dependence of the shock thickness on Alfven Mach number with the theoretical estimate based on wave interactions.

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FOOTNOTES

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\+ The same problem was discussed at the Fourth International Conference on Ionization Phenomena in Gases, Uppsala, August 1959. The present paper represents an amplification of the discussion and some minor changes in the theoretical treatment of the problem.

1 A. R. Kantrowitz and H. E. Petschek, "An Introductory Discussion of Magnetohydrodynamics" from Magnetohydrodynamics, edited by R. K. M. Landshoff (Stanford University Press, Stanford, California, 1957).

2 H. E. Petschek, Rev. Mod. Phys. 30, 966 (1958).

3 The characteristic ion Larmor radius for protons is defined in terms of an ion moving at the Alfvén speed and is $r_i = \sqrt{\frac{m_i c^2}{4 \pi N e^2}}$ where m_i is the proton mass, c is the velocity of light, N is the particle density and e is the electronic charge. The corresponding characteristic electron Larmor radius is smaller by the square root of the mass ratio.

4 F. D. Kahn, Gas Dynamics of Cosmic Clouds, Chap. 20, P. 115-116, "The Collision of Two Highly Ionized Clouds".

5 E. N. Parker, Phys. Rev. To be published.

6 C. S. Gardner, H. Goertzel, H. Grad, C. S. Morawetz, M. J. Rose and H. Rubin, Proc. 2nd U. N. Int. Conf. on Peaceful Uses of Atomic Energy, Vol. 31, 15/P/374.

7 The tacit assumption which has been made that the scattering of waves on waves is incoherent will be discussed in more detail below.

8 An important distinction is to be noted in this regard between this case and the case of ordinary aerodynamic turbulence. These two cases are, of course, similar in the sense that large scale motions lead to dissipation. The basic difference resides in the fact that the magnetohydrodynamic waves propagate through the fluid in the absence of other waves, while the turbulent eddies have no velocity relative to the fluid in the absence of other eddies. Therefore, while the magnetohydrodynamic wave moves away from its neighbors between collisions, the turbulent eddy does not. Correlations are therefore unimportant in the magnetohydrodynamic case but very important for ordinary turbulence. This makes the magnetohydrodynamic problem considerably more tractable theoretically.

9 The existence of a momentum transfer from the waves to the fluid or some form of friction between the wave field and the fluid is required to form a shock wave, since in the absence of such friction, there would be no coupling between the mean velocity of the wave distribution and the fluid. This relative velocity corresponds to a heat flux.

Since the amplification mechanism builds up principally waves with a large velocity relative to the fluid, in the absence of friction the corresponding heat flux would continue into the downstream region. This is incompatible with the Rankine-Hugoniot equations. At first sight, one might expect that interactions between waves would conserve momentum in the wave field. This difficulty can, however, be obviated by at least one mechanism. This is that interactions between fast waves could transfer momentum to the slow waves. Since the slow waves move almost at the fluid velocity and since these waves have appreciable damping, this may be considered effectively as a transfer of momentum to the fluid.

Interactions which determine the rate of change of the magnitude of k are not important in determining the transport coefficients of the gas. Their importance is associated with the dependence of the mean free path on the mean value of k, and the fact that the existence of very large values of k would give rise to appreciable damping of the waves. If one considers interaction of the waves by the nonlinear superposition of two waves, it is easily seen that the scattered wave must have a wave number and frequency which are the sum or difference of the wave numbers and frequencies of the superposed waves. Since the scattered wave must also satisfy the dispersion relation, only a limited group of waves can interact. In general, this limitation requires the initial wave vectors to be at a considerable angle to each other so that the magnitude of the resulting wave vector is not much larger than the initial ones. The order of magnitude of the mean wave vector will therefore not change appreciably

in the relatively few collisions required for a shock wave. Thus the wave mean free path is the one appropriate to waves of the wave number selected by the growth mechanism, and further, the wave field will not be damped through a diffusion to large k values.

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